CALCULATION OF CYLINDRICAL DETONATION WAVE EXPANDING FROM LINE OF EXPLOSION

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We examine the problem of explosion product motion behind an expanding cylindrically symmetric detonation wave propagating in a space filled with stationary explosive material ($u = 0$) with constant density $\rho_0 = \text{const}$ and $p_0 = 0$. We assume that: 1) the motion of the explosion products is isentropic; 2) the detonation wave corresponds to the Jouguet point of the Hugoniot adiabat.

The subject problem is solved analogously to the solution of the problem on propagation of an expanding spherically symmetric detonation wave suggested by Zel'dovich [1].

The motion of the explosion products behind a detonation wave front is described by the equations of gasdynamics, the equation of state, and the boundary conditions. We shall examine the motion in a plane perpendicular to the axis of symmetry. We introduce in this plane the r, φ polar coordinate system with origin at the point of intersection of the axis of symmetry and the given plane.

The equation of state is $p = A\rho^{\varkappa}$, where A is an arbitrary constant, $\varkappa$ is the adiabatic exponent, p is the pressure of the combustion products, and ρ is the density. We take as the unknowns the speed of sound c and the combustion product component velocity directed along the r axis. We denote it by u.

Knowing c, we can find p and ρ from the formulas

$$\rho = \left(\frac{c^2}{A\varkappa}\right)^{1/(\varkappa-1)}, \qquad p = A\left(\frac{c^2}{A\varkappa}\right)^{\varkappa/(\varkappa-1)}$$

The flow is bounded on one side by the free surface ($p = 0$) and on the other side by the detonation wave front. The values of the functions at the front must satisfy the conditions

$$\rho_1 D = \rho_2 (D - u_2), \qquad \rho_1 D^2 = \rho_2 (D - u_2)^2 + p_2, \qquad D = u_2 + c \tag{1}$$

where D is the detonation wave propagation velocity. The subscript 1 indicates values of the functions ahead of the wave; 2 denotes values of the functions behind the wave.

The motion of the explosion products is self-similar. Therefore all the gasdynamic quantities are functions of the variable $\zeta = r/t$. After converting in the gasdynamic equations to the variable ζ we obtain the system of ordinary differential equations

$$\left[\frac{(\zeta - u)^2}{c^2} - 1\right]\frac{du}{d\zeta} = \frac{u}{\zeta}, \qquad (\zeta - u)\frac{du}{d\zeta} = \frac{2c}{\varkappa - 1}\frac{dc}{d\zeta} \tag{2}$$

From the boundary conditions (1) we have for the value of the unknown functions at the detonation wave front

$$\zeta = D = (1 + \varkappa) R, \qquad u = R, \qquad c = \varkappa R \tag{3}$$

where

$$R = [A\varkappa^{-\varkappa}((1 + \varkappa)\rho_1)^{\varkappa - 1}]^{1/2}$$

By the variable replacement

$$u = Ru^*, \qquad \zeta = R\zeta^*, \qquad c = Rc^* \tag{4}$$

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the boundary conditions (3) can be brought to the form

$$u^* = 1, \qquad \zeta^* = \varkappa + 1, \qquad c^* = \varkappa \tag{5}$$

The system (2) is invariant with respect to the variable change (4).

Thus the problem is reduced to integration of (2) with the boundary conditions (5).

As in the case of spherically symmetric detonation wave propagation, study of (2) shows that the motion of the explosion products takes place in the region $c_0^* \leq \zeta^* < \varkappa + 1$, where c_0^* is the value of the function c^* for $u^* = 0$.

In the vicinity of the point ($\zeta^* = c_0^*$, $c^* = c_0^*$, $u^* = 0$) the solution has the form

$$\zeta^* = c_0^* + (\varkappa + 1)\, u^* + A_1 (u^*)^2, \qquad c^* = c_0^* + {}^1/_2 (\varkappa - 1)\, u^* \tag{6}$$

Here A_1 is an arbitrary constant.

We see from (6) that the aft boundary of the motion region will be a weak discontinuity surface. At this surface $du^*/d\zeta^*$ undergoes a discontinuity. The propagation velocity of this boundary equals the speed of sound. The front boundary of the motion region will be the detonation wave front surface. In the vicinity of this surface the solution has the form

$$\zeta^* = (\varkappa + 1) - {}^1/_2 \varkappa^{-1} (\varkappa + 1)^2 (u^* - 1)^2, \qquad c^* = \varkappa + {}^1/_2 (\varkappa - 1)(u^* - 1)$$

The system (2), (5), was solved on a computer for the adiabatic exponents $\varkappa = 1.666...$, 2, 2.5, 3.

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LITERATURE CITED

1. Ya. B. Zel'dovich, "On the pressure and velocity distributions in the products of a detonative explosion, specifically for spherical propagation of the detonation wave," ZhETF, Vol. 12, No. 9 (1942).